INSTABILITY IN A GRANULAR BED FLUIDIZED BY A GAS FLOW

V. A. Borodulya, Yu. A. Buevich, and V. V. Zav'yalov

UDC 532.529.5

Stability in fluidization is considered in an approach in which the granular bed is considered as a structureless element with a definite working response, and the boundary to the stability region in parameter space is examined.

Previous studies of stability in fluidized systems amount essentially to analysis of the purely hydrodynamic behavior of small perturbations in the porosity and other quantities that describe the homogeneous fluidized state [1-8]. Such a state is always unstable, since any fluidized bed has perturbations that increase with time, and the various layers in that respect differ only in their values for the characteristic rate of growth of the perturbations. * If the rate of growth is small, the perturbations remain small even on emergence at the upper boundary of the bed, and the structure of the bed does not apparently differ from homogeneous, whereas at high rates of increase, which are characteristic particularly of fluidization by gases, one gets essentially nonlinear interactions between the perturbations of various wavelengths and frequencies, which result in the generation of finite-amplitude waves [9, 10], with the ultimate result being gas bubbles and other discontinuities, the bed thus becoming nonuniformly fluidized.

However, in both cases the conclusions derived from such an analysis apply only to the internal structure of the fluidized bed, and they have no relationship to the observed global behavior. In particular, these conclusions cannot be utilized directly to evaluate the behavior of macroscopic characteristics such as the effective height of the fluidized bed, the overall pressure difference, and so on. The type of equipment and the working

*It follows from [7, 8] that the internal pressure in the dispersed phase due to the random fluctuations of the particles can sometimes (especially when the concentration of the dispersed phase is small) stabilize the uniform state and suppress small perturbations. However, no matter how effective such stabilization for dilute suspensions, it is unimportant for realistic values of the porosity and other quantities commonly occurring during fluidization.

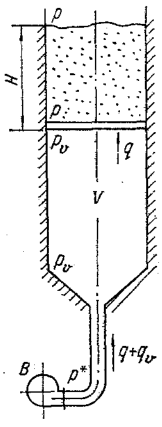


Fig. 1. The equipment.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Institute of Problems of Mechanics, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 3, pp. 410-417, September, 1976. Original article submitted November 13, 1975.

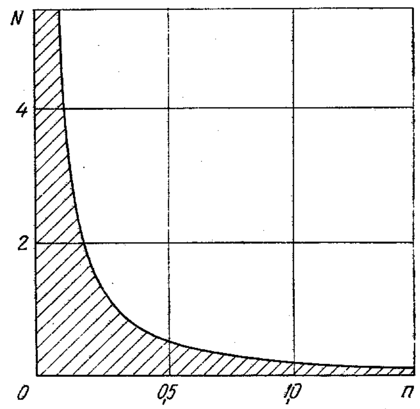


Fig. 2. Unstable region in fluidization in the (n, N) plane.

parameters may result in these characteristics being stable on average (neglecting possible random fluctuations due to local nonuniformities within the bed) or else showing regular periodic pulsations due to some global instability which, in general, is not related to instability in the internal structure of the bed itself. Examples of such pulsations are ones seen in the pressure in beds fluidized by gases or even by liquids [11, 12], as well as the relaxation oscillations observed in granular beds [13, 14].

From the purely practical viewpoint, a study of stability in fluidization in the above sense is of no less interest than the traditional stability analysis, examination of the conditions for nonuniform fluidization, and so on. So far as we are aware, no study has previously been made of the stability in the bed depth, pressure difference, and other such macroscopic characteristics, apart from isolated attempts to describe the dynamic response to such quantities for a homogeneous fluidized bed to sudden changes in the fluidization conditions [15, 16].

Figure 1 shows a schematic model that reflects the major features of real equipment, the basic parameter being the current height H of the bed, which serves to characterize the fluidization generally.

The gas is supplied at an initial pressure p^* and enters the free space V, where the pressure is p_V; the total mass flow rate is $q + q_V$. Part of the gas (flow rate q) passes through the gas distributor to the granular bed and fluidizes it; the pressure differences in the distributor and in the bed are, respectively, $p_V - p$ and $p - p^0$. The treatment is simplified by assuming that the gas is ideal, while the pressure differences in the supply system and in the distributor (grid) are linearly dependent on the corresponding flow rates, and the pressure difference across the bed is small by comparison with the external pressure p^0, which enables us to neglect the variation in gas density over the height of the bed. It is clear that these assumptions are not fundamental and correspond to the following relations:

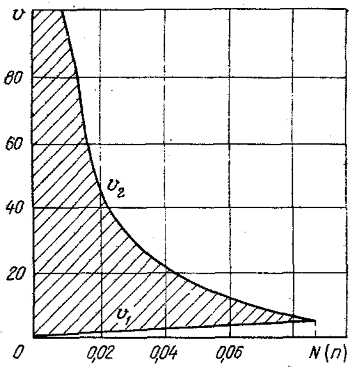
$$\Delta_1 p = p^* - p_v = k_1 (q + q_v),\ \Delta_2 p = p_v - p = k_2 q,\ \Delta_3 p = p - p^0 \ll p^0,$$
$$p = \rho \frac{RT}{M} = \frac{\rho}{c},\quad c = \frac{M}{RT}. \tag{1}$$

Also, we assume that the bed is macroscopically homogeneous in that the density averaged over the cross section is dependent only on the instantaneous value of H (i. e., on time), being independent of the position of the section in the bed. This represents some degree of idealization for the passage of gas through the bed, but it is reasonably close to the actual position for many fluidized beds of granular material. The opposite assumption would perhaps be that the granular bed moves as a whole in the fashion of a piston.

The basic equation for the behavior of a macroscopically homogeneous bed is put as

$$\frac{1}{2} m\ddot{H} + mg = \Delta_3 p = p - p^0,\ m = \rho_b H_0, \tag{2}$$

where a dot above a symbol represents differentiation with respect to time. This equation contains no term for the friction at the wall, because the frictional stress at the wall is proportional to the pressure of the bed on the wall, which vanishes when the bed is fluidized (stresses due to momentum transfer by fluctuating particles are comparatively small and therefore are neglected).



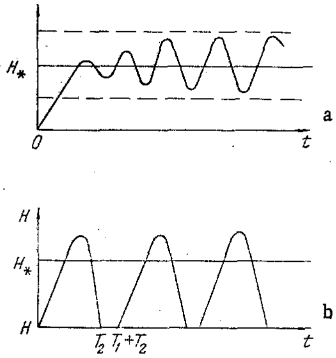


Fig. 3 Fig. 4

Fig. 3. The unstable region in terms of the variables N and v for n = 1.

Fig. 4. Establishment of: a) an ordered oscillatory state; b) relaxation oscillations.

From (1) and (2) we have

$$\frac{1}{2}m\ddot{H} + mg + (k_1 + k_2)q + k_1 q_v = \Delta p = p^* - p^0, \tag{3}$$

in which the right side may be considered in the general case as some given function of time. It is clear that to close (3) we need to express q_V and q as functions of H and derivatives of the latter. We assume that p^0 is constant, to get from (1) and (2) together with the definition of q_V that

$$q_v = V\dot{\rho}_v = cV\dot{p}_v = cV\frac{d}{dt}(\Delta_2 p + \Delta_3 p) = cV\left(\frac{m}{2}\dddot{H} + k_2\dot{q}\right). \tag{4}$$

Then (3) may be closed by deriving a unique function, namely, the working characteristic $q(H, \dot{H}, \ldots)$ of the fluidized bed. We avoid discussing the details of the local structure in the bed insofar as this does not affect the derivation of this characteristic, which is entirely reasonable here. A similar approach is widely used in the theory of mechanical and electrical oscillations.

Of course, the form of this function is dependent on the type of fluidized bed; for definiteness, we restrict consideration to the state of developed inhomogeneous fluidization on the basis that the assumptions of the two-phase fluidization theory are applicable. That is, we assume that a proportion δ of the total volume is occupied by rising gas bubbles, which are virtually free from particles, while the rest of the volume is filled by a compact phase in a state of minimum fluidization. We calculate the mean volumetric gas flow rate to the upper boundary of the bed and equate this to the gas flow from the distributor to get

$$Q = Q_0 + \dot{H} + \delta(Q_b - Q_0),\quad \delta = \frac{H - H_0}{H}, \tag{5}$$

where we have used an obvious representation for δ; note that the volumetric speed of the gas in the accelerated compact phase is different from Q_0, but the difference is of minor importance if $\ddot{H} \ll g$, which is usually the case.

From (5) we get the desired working characteristic as

$$q = q_0 + \rho S\dot{H} + \sigma\frac{H - H_0}{H},\quad q_0 = \rho S Q_0,\quad \sigma = \rho S(Q_b - Q_0), \tag{6}$$

which goes with (3) and (4) to close the system of equations for this system as regards the response to external perturbations.†

† Note that (5) or (6) alone allows us to solve an extremely important independent problem, namely, the bed expansion consequent on a given Q(t), which previously has been considered fully only for a uniformly fluidized bed [15, 16]. The solution takes a particularly simple form for the case $H - H_0 \ll H_0$. We assume that the speed of the gas in the bubble phase is constant, in which case we have from (5) that

$$H(t) \approx H_0 + (H(Q) - H_0)\exp\left(-\frac{Q_b - Q_0}{H_0}t\right) + \int_0^t (Q(\tau) - Q_0)\exp\left[-\frac{Q_b - Q_0}{H_0}(t - \tau)\right]d\tau.$$

A disadvantage of this model is that the mean gas speed in the bubble phase Q_b is taken as independent of the degree of expansion of the bed, and thus of time; however, that assumption follows directly from the assumption that the two-phase fluidization theory is applicable, and it would appear that the two assumptions are comparable in justification. In the more general case, one should incorporate possible deviations from the two-phase theory (see the discussion in [17]) and take Q_b as a slowly varying function of time. For our purpose it is sufficient to assume that Q_b and σ in (6) are certain constant characteristics of a given bed.

We assume that Δp = const to get the following steady-state solution for H_* and q_* to (3), (4), and (6):

$$H_* = \frac{\sigma}{\sigma - q_* + q_0} H_0 = \frac{Q_b - Q_0}{Q_b - Q_*} H_0,$$
$$q_* = \rho S Q_* = \frac{\Delta p - mg}{k_1 + k_2} \tag{7}$$

(it is clear that the existence condition for a steady state of fluidization is provided by the inequality $\Delta p > mg$).

To consider the stability of the state of (7) we examine states deviating slightly from this: from (3)-(7) we get for such states that

$$\frac{1}{2} m\ddot{x} + (k_1 + k_2)(q - q_*) + cVk_1 \left(\frac{1}{2} m \dddot{x} + k_2 \dot{q}\right) = 0,$$
$$q = q_* + \rho S(\dot{x} + \nu x),\ x = H - H_*,\ \nu = H_0 H_*^{-2}(Q_b - Q_0), \tag{8}$$

which gives us the following equation for the small oscillations:

$$\dddot{x} + a_2\ddot{x} + a_1\dot{x} + a_0 x = 0,\ a_0 = \frac{2\nu\rho S(k_1 + k_2)}{mk_1 cV},$$
$$a_1 = 2\rho S \frac{k_1 + k_2 + \nu k_1 k_2 cV}{mk_1 cV},\ a_2 = \frac{m + 2\rho k_1 k_2 cVS}{mk_1 cV}. \tag{9}$$

This equation has undamped solutions, i. e., the state of (7) is unstable if the corresponding characteristic equation has even one root with a positive real part, i. e., if $a_1 a_2 < a_0$; simple transformation converts this condition to the following condition for the instability of the steady state of fluidization in the presence of small perturbations:

$$(cV)^2 + 2\alpha cV + \beta < 0,$$
$$2\alpha = \frac{2\rho S k_1 k_2 (k_1 + k_2) - \nu m k_1^2}{2\nu\rho S k_1^2 k_2^2},\ \beta = \frac{m(k_1 + k_2)}{2\nu\rho S k_1^2 k_2^2}. \tag{10}$$

It is convenient to introduce here the dimensionless volume v of the free space and the dimensionless parameters N and n by means of

$$v = \nu c k_1 V,\ N = \frac{2\rho S k_1}{\nu m},\ n = \frac{k_2}{k_1}. \tag{11}$$

Then (10) is put as

$$L(v) = v^2 + \left[1 + n - \frac{1}{Nn}\right]\frac{v}{n} + \frac{1 + n}{Nn^2} < 0. \tag{12}$$

It is readily seen that this condition is not satisfied, i. e., the steady state is stable for v very small and for v large; however, it can be satisfied for a certain range in v if the equation L(v) = 0 has two real positive roots. The latter occurs, as is readily shown, if N and n satisfy

$$1 + n - \frac{1}{Nn} < -2\left(\frac{1 + n}{N}\right)^{\frac{1}{2}} \tag{13}$$

The instability region in the (N, n) plane is shown by the hatching in Fig. 2; the equation for the boundary of this region is

$$N = \frac{1}{n(1 + n)} \cdot \frac{1}{1 + 2n + \sqrt{(1 + 2n)^2 - 1}}. \tag{14}$$

If (12) or (13) is met, the stationary fluidization state actually is unstable if v lies within the range (v_1, v_2), where

$$v_{1,2}=\frac{1}{2n}\left[\frac{1}{Nn}-1-n\right]\left\{1\pm\left[1-\frac{4}{N}\frac{1+n}{[(Nn)^{-1}-1-n]^2}\right]^{\frac{1}{2}}\right\}. \tag{15}$$

As an example, Fig. 3 (hatched region) shows the instability in the (N, v) plane, which is bounded by the curves of (15) for n = 1. Such regions are readily constructed for other values of n, and these regions move toward higher v as n increases, i. e., upward in Fig. 3.

These results allow us to examine the unstable region in relation to the various physical and other parameters; for instance, (7), (8), and (11) give

$$N=2\frac{\rho}{\rho_b}\left(\frac{Q_b-Q_0}{Q_b-Q_*}\right)^2\frac{k_1S}{Q_b-Q_0}. \tag{16}$$

This shows directly that the unstable range (v_1, v_2) is independent of H_0 for any given n, and also that instability sets in more readily as ρ/ρ_b, S, and Q_* decrease. The effects of the coefficients k_1 and k_2 on the stability are more complicated. For instance, increase in k_1 and k_2, on the one hand, tends to stabilize the process for given n and Δp, since there is a proportional increase in N, whereas, on the other hand, there is a destabilizing effect, since there is a reduction in Q_*, as defined by (7), and thus a corresponding fall in N. Similarly, the critical volumes V_1 and V_2 for the cavity corresponding to the onset of instability are as follows in terms of the parameters:

$$V_{1,2}=\frac{H_0}{ck_1(Q_b-Q_0)}\left(\frac{Q_b-Q_0}{Q_b-Q_*}\right)v_{1,2}, \tag{17}$$

which follows from (7), (8), and (11) via the $v_{1,2}$ of (15).

The increments of the oscillations and the frequencies of the growing perturbations are found as the real and imaginary parts of the roots λ of the characteristic equation corresponding to (9); we introduce the parameters of (11) to get from (9) that

$$\left(\frac{\lambda}{\nu}\right)^3+\frac{1+vNn}{v}\left(\frac{\lambda}{\nu}\right)^2+N\frac{1+n+vn}{v}\left(\frac{\lambda}{\nu}\right)+\frac{1+n}{vN}=0. \tag{18}$$

This shows that the frequencies and increments are proportional to the ν given by (8).

In conclusion, we note briefly some possible consequences of instability in the steady state; the linear equations (8) and (9) cease to be suitable as the perturbations become larger, and one then has to use the initial nonlinear system of (3), (4), and (6); the nonlinearity halts the growth of the perturbations when a certain state is reached, so a steady oscillation amplitude is set up. A preliminary analysis indicates that mild oscillatory conditions arise on passing through the stability boundary in parameter space; i. e., the amplitude increases monotonically from zero as the image point moves across the stability boundary into the unstable region. In that case one gets an ordered oscillatory state, which is similar to the secondary flows encountered in hydrodynamics. Figure 4a shows schematically how this state might be reached. The fluctuations in H may exceed H_*-H_0 as the deviation from the critical state increases, and in that case the bed returns to the immobile state after some time T_2 after the start and remains in that state for some time T_1, after which the process repeats (Fig. 4b). The first of these states corresponds to almost harmonic oscillation, as has been observed [11, 12], while the second relates to relaxation oscillations [13, 14]. Both states are of independent interest, but an examination of these falls outside the scope of this paper.

NOTATION

c, quantity defined in (1); g, acceleration due to gravity; H, bed height; H_0, H_*, bed heights in immobile and steady fluidized states, respectively; k_1, k_2, resistance coefficients of the gas supply system and distributor; m, bed mass per unit cross-sectional area; M, molecular weight; N, n, dimensionless parameters in (11); p^*, p^0, p_V, p, pressures at the inlet, at the outlet, in a free gas cavity, and at the outlet of gas distributor; Q, gas flow rate; Q_0, Q_*, Q_b, minimum fluidization speed, gas flow speed, in stationary fluidized state, and mean gas velocity in bubble phase; q, q_V, total mass flow rates into bed and into free cavity; R, gas constant; S, bed cross-sectional area; V, volume of cavity under distributing grid accessible to gas; v, dimensionless volume in (11); x, deviation of bed height from constant value; δ, proportion of bubble phase by volume in fluidized bed; ν, parameter in (8); ρ, gas density; ρ_b, bulk density of immobile bed; σ, coefficient from (6).

LITERATURE CITED

1. R. Jackson, Trans. Inst. Chem. Eng., 41, T13 (1963).
2. R. L. Pigford and T. Baron, Ind. Eng. Chem. Fund., 4, 81 (1965).
3. J. D. Murray, J. Fluid. Mech., 21, 465 (1965).
4. O. Molerus, in: Proceedings of the International Symposium on Fluidization, Amsterdam, Netherlands University Press (1967).
5. J. Van Deemter, in: Proceedings of the International Symposium on Fluidization, Amsterdam, Netherlands University Press (1967).
6. E. Ruckenstein and M. Tzeculescufilipescu, Chem. Eng. Sci., 23, 1121 (1968).
7. T. B. Anderson and R. Jackson, Ind. Eng. Chem. Fund., 7, 13 (1968).
8. Yu. A. Buyevich, J. Fluid Mech., 56, 313 (1972).
9. T. B. Anderson and R. Jackson, Ind. Eng. Chem. Fund., 8, 137 (1969).
10. J. Verloop, P. M. Heertjes, and L. A. Lerk, Chem. Eng. Sci., 29, 1109 (1974).
11. A. A. Drinkenburg, in: Proceedings of the International Symposium on Fluidization, Amsterdam, Netherlands University Press (1967).
12. J. Verloop and P. M. Heertjes, Chem. Eng. Sci., 29, 1035 (1974).
13. H. W. Wong and M. H. I. Baird, Chem. Eng. J., 2, 104 (1971).
14. M. H. I. Baird and A. J. Klein, Chem. Eng. Sci., 28, 1039 (1973).
15. P. L. Slis, T. W. Willemse, and H. Kramers, Appl. Sci. Res., Sec. A, 8, 209 (1959).
16. L.-T. Fan, J. A. Schmitz, and E. N. Miller, AIChE J., 9, 149 (1963).
17. J. R. Grace and R. Clift, Chem. Eng. Sci., 29, 327 (1974).

KINETIC EQUATIONS OF HIGH-INTENSITY HEAT AND MASS TRANSFER

P. P. Lutsik

UDC 66.047.35

A system of basic kinetic equations of high-intensity heat and mass transfer in capillary-porous bodies is obtained by averaging the equations of heat and mass transfer with variable coefficients.

The use of averaged relations between quantities obtained on the basis of the equations of heat and mass transfer is a useful method of generalizing experimental data and developing engineering methods for the calculation of heat- and mass-transfer processes. This "integral approach" was adopted in [1] to find the dependence of the heat flow on the rate of drying and heating of a body, and was further developed in [2] and elsewhere. It might be expected that an analogous approach would be just as useful in more general and more complex problems.

In the present work, this approach is extended to the case of high-intensity heat and mass transfer, in which filtrational mass transfer begins to play a significant role. This allows kinetic equations to be obtained for the heat flow $j_q(\tau)$ and the mass flow of material leaving the body — the total flow $j_m(\tau)$ and its filtrational (molar) component $j_p(\tau)$. The problem is solved without any assumptions as to the constancy of the coefficients of heat and mass transfer or the kinds of contact between the material and the surrounding medium.

Intensive heat and mass transfer in a capillary-porous body is described by a system of nonlinear equations [1, 3]:

$$\frac{\partial T}{\partial \tau} = \nabla (a_q \nabla T) + \frac{\varepsilon r}{c_q} \frac{du}{d\tau} , \tag{1}$$

Kiev Technological Institute of the Light Industry. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 3, pp. 418-422, September, 1976. Original article submitted July 23, 1975.